

Teorem:(A-G nejednakost)

Ako su  $a$  i  $w$  uređene  $n$ -torke pozitivnih realnih brojeva, tada vrijedi  $A_n(a, w) \geq G_n(a, w)$ .

Neka je  $a = (a_1, a_2, \dots, a_n)$ ,  $w = (w_1, w_2, \dots, w_n)$ , te  $W := w_1 + w_2 + \dots + w_n$ .  
 . Neka je  $0 < a_1 \leq a_2 \leq \dots \leq a_n$ , tada je  $\frac{a_1}{a_i} \leq 1, i = 1, 2, \dots, n$ , te također  $\frac{a_i}{a_n} \leq 1, i = 1, 2, \dots, n$ .

Odavde dobivamo,  $\frac{a_1}{a_n}w_1 + \frac{a_2}{a_n}w_2 + \dots + \frac{a_n}{a_n}w_n \leq w_1 + w_2 + \dots + w_n$  tj.  $\frac{1}{a_n}(a_1w_1 + a_2w_2 + \dots + a_nw_n) \leq w_1 + w_2 + \dots + w_n$ ,  $A_n(a, w) = \frac{a_1w_1 + a_2w_2 + \dots + a_nw_n}{w_1 + w_2 + \dots + w_n} \leq a_n = \max\{a_1, a_2, \dots, a_n\}$ .

Analogno se dobije  $\min\{a_1, a_2, \dots, a_n\} = a_1 \leq \frac{w_1 + w_2 + \dots + w_n}{\frac{w_1}{a_1} + \frac{w_2}{a_2} + \dots + \frac{w_n}{a_n}} = H_n(a, w)$ .

Imamo, dakle  $\frac{\frac{1}{a_1}w_1 + \frac{1}{a_2}w_2 + \dots + \frac{1}{a_n}w_n}{w_1 + w_2 + \dots + w_n}$ , primjenimo A-G nejednakost na brojeve  $\frac{1}{a_1}, \frac{1}{a_2}, \dots, \frac{1}{a_n}$ , dobivamo  $(\frac{1}{a_1} * \frac{1}{a_2} * \dots * \frac{1}{a_n})^{\frac{1}{W}} \leq \frac{\frac{w_1}{a_1} + \frac{w_2}{a_2} + \dots + \frac{w_n}{a_n}}{w_1 + w_2 + \dots + w_n}$ , tj.  $H_n(a, w) = \frac{w_1 + w_2 + \dots + w_n}{\frac{w_1}{a_1} + \frac{w_2}{a_2} + \dots + \frac{w_n}{a_n}} \leq (a_1^{w_1} a_2^{w_2} \dots a_n^{w_n})^{\frac{1}{W}} = G_n(a, w)$ .

Dakle,  $\min\{a_1, a_2, \dots, a_n\} \leq H_n(a, w) \leq G_n(a, w) \leq A_n(a, w) \leq \max\{a_1, a_2, \dots, a_n\}$